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VUV and X-ray Free-Electron Lasers

Time-independent 3D FEL Simulations

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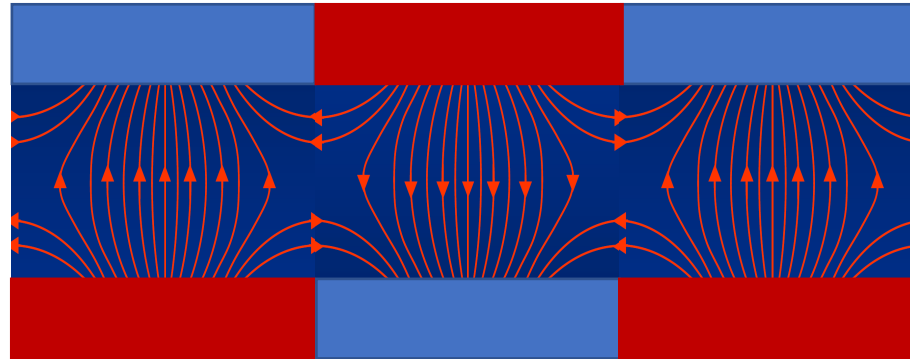
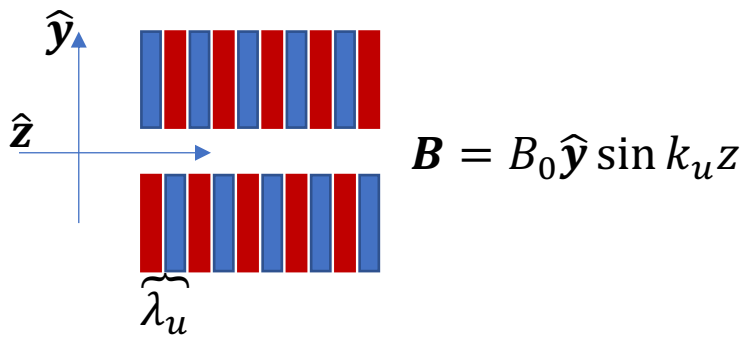


Thursday (Jan 28) Lecture Outline

- Transverse dynamics and natural focusing
- Q&A
- Strong focusing and a FODO lattice
- Q&A
- Ming Xie analysis
- Q&A
- Numerical simulator – LUME-Genesis
- Q&A

Transverse dynamics and natural focusing

Planar Undulator

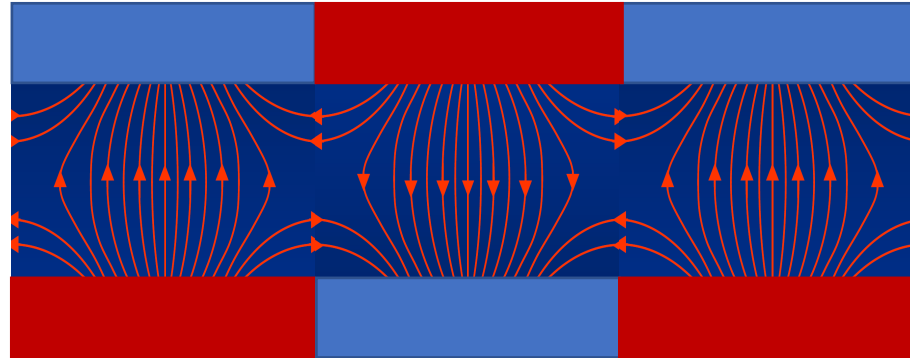
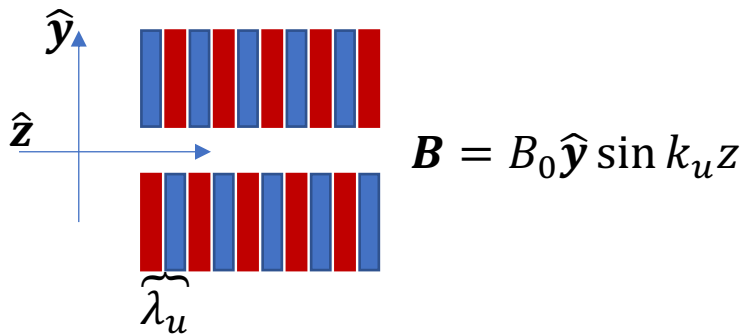


$$\mathbf{B} = B_0 \hat{\mathbf{y}} \sin k_u z \cosh k_u y + B_0 \hat{\mathbf{z}} \cos k_u z \sinh k_u y$$

- A previously assumed magnetic field does not satisfy $\nabla \cdot \mathbf{B} = 0$!
- The vertical sinusoidal field that satisfies Maxwell's equation has the vector potential $A_x = -\frac{B_0}{k_u} \cos k_u z \cosh k_u y$ that results in $B_z \neq 0$!
- Lorentz force (written as the second derivative with respect to z) results in:
 - $x'' = k_u \frac{K}{\gamma} \cosh k_u y \sin k_u z$, which is a standard FEL equation (2.7) for $y \approx 0$
 - $y'' + k_u \frac{K^2}{\gamma^2} \cos^2 k_u z \frac{\sinh 2k_u y}{2} = -p_x k_u \frac{K}{\gamma} \sinh k_u y \cos k_u z \approx 0$

According to S.Y. Lee *Accelerator Physics*, 4th ed, chapter 4.III.2

Planar Undulator



- The nonlinear magnetic field can be neglected since $k_u y \ll 1$:
 - $y'' + \left(k_u^2 \frac{K^2}{\gamma^2} \cos^2 k_u z \right) y = 0$
- Many FEL codes assume that dynamics is **averaged over the undulator period**:
 - $y'' = - \left\langle k_u^2 \frac{K^2}{\gamma^2} \cos^2 k_u z \right\rangle_u y = -k_u^2 \frac{K^2}{2\gamma^2} y$, which correspond to “natural focusing” of the undulator and the Hill’s equation $y'' + K_p y = 0$, with the focusing function $K_p = k_u^2 \frac{K^2}{2\gamma^2}$
- The helical undulator has $K_x = K_y = \frac{1}{\sqrt{2}} K_p$

Matrix transformation

- Solution of the Hill's equation for the planar undulator is

$$\begin{bmatrix} y \\ y' \end{bmatrix}_{z \rightarrow z+L} = \begin{bmatrix} \cos \frac{K}{\sqrt{2}\gamma} k_u L & \frac{\sin \frac{K}{\sqrt{2}\gamma} k_u L}{\frac{K}{\sqrt{2}\gamma} k_u} \\ -\frac{K}{\sqrt{2}\gamma} k_u \sin \frac{K}{\sqrt{2}\gamma} k_u L & \cos \frac{K}{\sqrt{2}\gamma} k_u L \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}_z$$

Matrix transformation

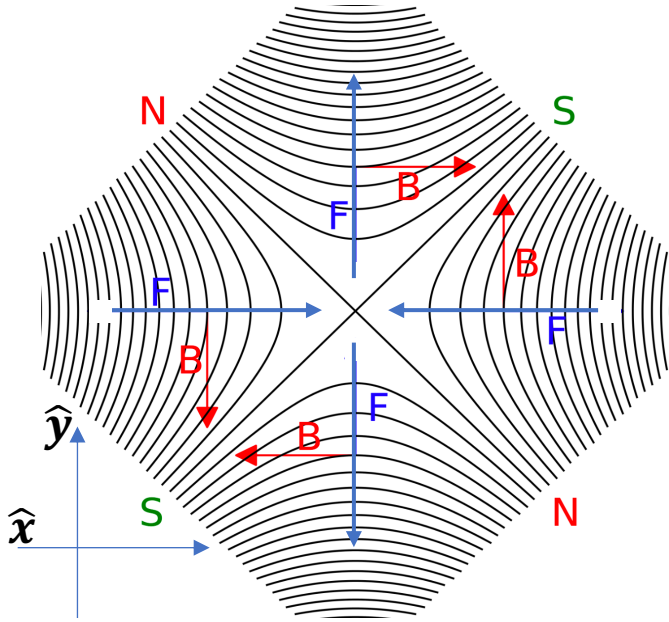
- Solution of the Hill's equation for the planar undulator is

$$\begin{bmatrix} y \\ y' \end{bmatrix}_{z \rightarrow z+L} \approx \begin{bmatrix} 1 - \frac{K^2}{4\gamma^2} k_u^2 L^2 & L \\ -\frac{K^2}{2\gamma^2} k_u^2 L & 1 - \frac{K^2}{4\gamma^2} k_u^2 L^2 \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}_z$$

- This matrix transformation corresponds to a thick lens of the thickness L and the focal length $f = \frac{2\gamma^2}{K^2 k_u^2 L} = \frac{6825[m^2]}{L[m]}$ for MaRIE-like x-ray FEL;
- The natural focusing is not sufficient for controlling transverse beam dynamics in FELs.

Strong focusing and FODO lattice

Focusing in Quadrupole



$\mathbf{B} = B_1(y \hat{x} + x \hat{y})$, where $B_1 = \frac{\partial B_y}{\partial x} \left[\frac{T}{m} \right]$ used by Genesis

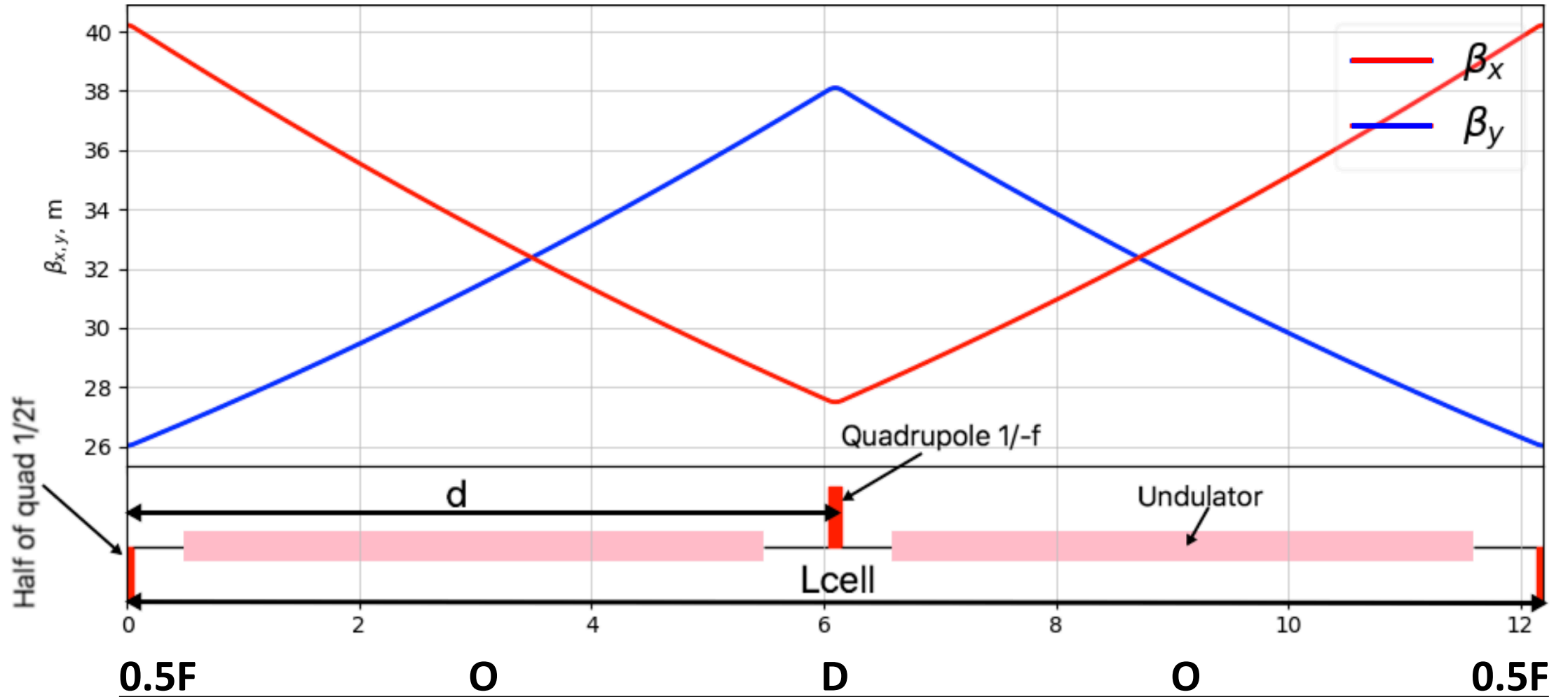
in order to calculate the focusing strength: $k[m^{-2}] = 0.299 \frac{B_1 \left[\frac{T}{m} \right]}{E[GeV]}$
with a sign-convention $k > 0$ for horizontal focusing ('QUADF');

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \rightarrow z+l} = \begin{bmatrix} \cos \sqrt{k}l & \frac{\sin \sqrt{k}l}{\sqrt{k}} \\ -\sqrt{k} \sin \sqrt{k}l & \cos \sqrt{k}l \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z, \text{ focusing plane}$$

$$\begin{bmatrix} y \\ y' \end{bmatrix}_{z \rightarrow z+l} = \begin{bmatrix} \cosh \sqrt{k}l & \frac{\sinh \sqrt{k}l}{\sqrt{k}} \\ \sqrt{k} \sinh \sqrt{k}l & \cosh \sqrt{k}l \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}_z, \text{ defocusing plane}$$

In the limit of $\sqrt{k}l \ll 1$, a thin quadrupole represents a thick lens of the length l and the focal length $f = (kl)^{-1}$.

FODO lattice – focusing in both planes



FODO lattice in thin lens approximation

- A single FODO cell in matrix notations for x coordinate is

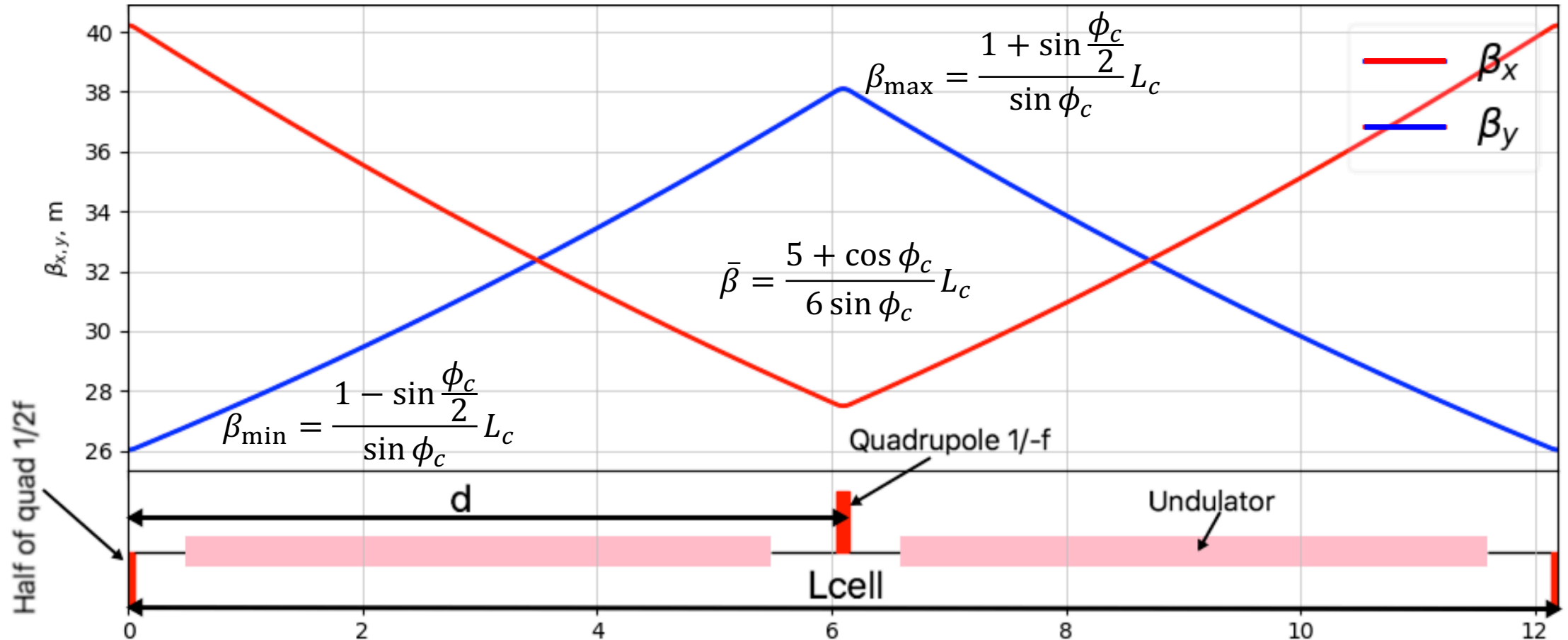
$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \rightarrow z+L_c} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{L_c}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{L_c}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

that simplifies to

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \rightarrow z+L_{cell}} = \begin{bmatrix} 1 - \frac{L_c^2}{8f^2} & L_c \left(1 + \frac{L_c}{4f} \right) \\ -\frac{L_c}{4f^2} \left(1 - \frac{L_c}{4f} \right) & 1 - \frac{L_c^2}{8f^2} \end{bmatrix} = U^T \begin{bmatrix} e^{i\phi_c} & 0 \\ 0 & e^{-i\phi_c} \end{bmatrix} U \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

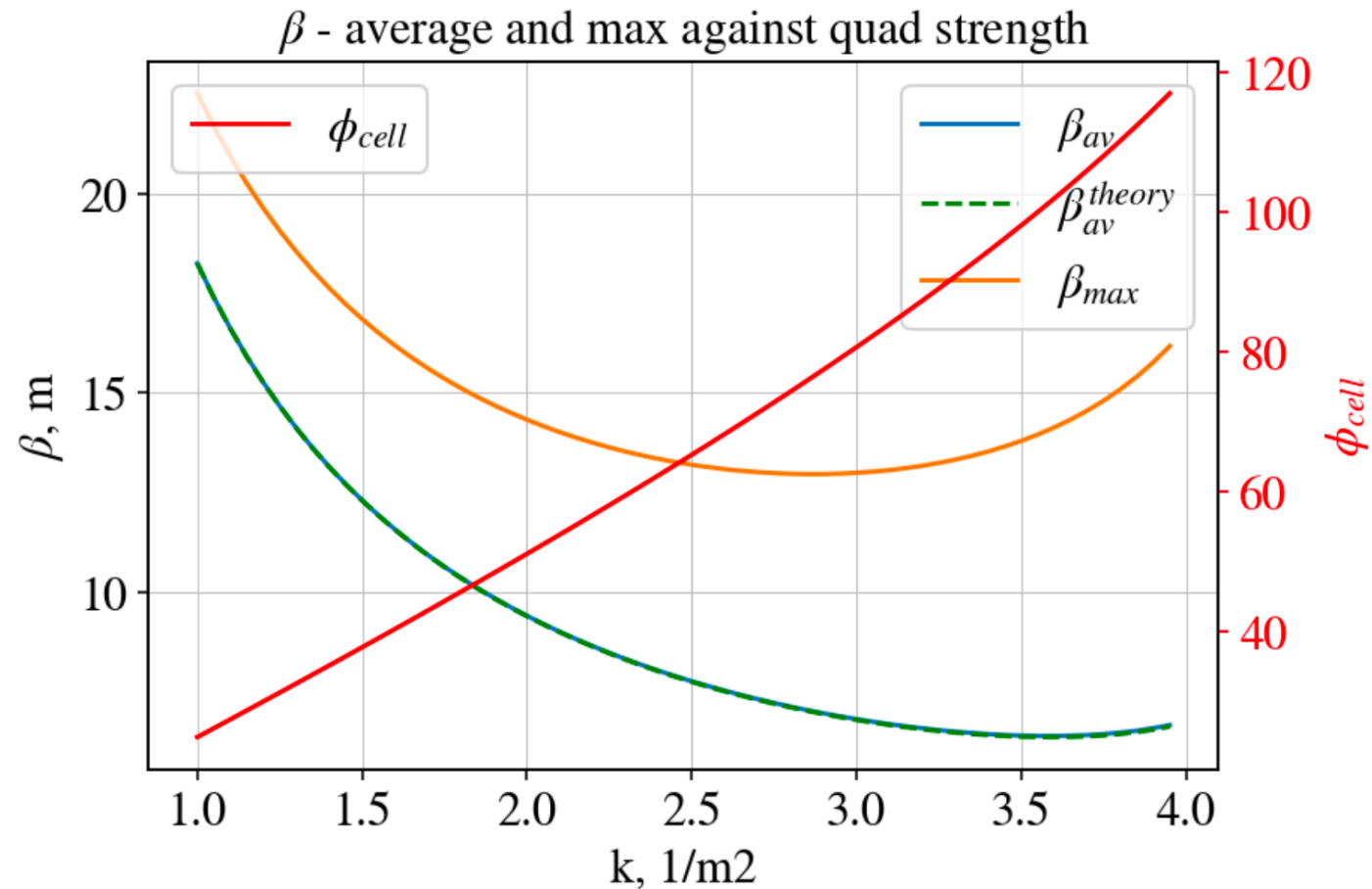
- The matrix transformation for y coordinate requires $f \rightarrow -f$ in the Eq. above;
- Phase advance is related to the transfer matrix by $\sin \frac{\phi_c}{2} = \frac{L_c}{4|f|}$, which results in a real phase and a stable solution if $|f| > L_c/4$.

FODO lattice in thin lens approximation



The minimum possible average $\bar{\beta}_{min}$ gives us $\phi_{cell} = \pi - \arctan 2\sqrt{6} \approx 101^\circ$

Beta function study for MaRIE-like x-ray FEL



Courant-Snyder Parametrization

- The most general form for matrix M_{FODO} with *unit modulus* can be parametrized as

$$M_{FODO} = \begin{bmatrix} \cos \phi_c + \alpha \sin \phi_c & \beta \sin \phi_c \\ -\gamma \sin \phi_c & \cos \phi_c - \alpha \sin \phi_c \end{bmatrix} = \mathbf{I} \cos \phi_c + \mathbf{J} \sin \phi_c ,$$

where α , β and γ are Courant-Snyder parameters of the periodic solution, ϕ_c is the phase advance, \mathbf{I} is the unit matrix, and

$$\mathbf{J} = \begin{bmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{bmatrix}, \text{ with } \text{Tr}(\mathbf{J}) = 0, \mathbf{J}^2 = -\mathbf{I} \text{ or } \beta\gamma = 1 + \alpha^2$$

- One can thus solve for FODO matched beam parameters;
- Using the property of matrix , we obtain the De Moivre's theorem:

$$M^n = \mathbf{I} \cos n\phi_c + \mathbf{J} \sin n\phi_c \text{ and } M^{-1} = \mathbf{I} \cos \phi_c - \mathbf{J} \sin \phi_c$$

that describe an oscillation if the beam parameters are not FODO matched.

Genesis Description

FODOlattice.ipynb Input parameters

```
# UNDULATOR parameters
XLAMD = 0.0186 # undulator wavelength
ku = 2*np.pi/XLAMD
AW0 = 0.86 # rms Undulator parameter
K = np.sqrt(2)*AW0
# FODO parameters
F1ST = 5
QUADF = 30
FL = 10
QUADD = 30
FD = 10

DRL = 100
# Beam parameters
GAMMA0 = 12e9/0.511e6 # beam energy in mc2
```

http://genesis.web.psi.ch/Manual/parameter_focusing.html
http://genesis.web.psi.ch/Manual/parameter_undulator.html
http://genesis.web.psi.ch/Manual/parameter_beam.html

Derived parameters

```
# Focusing quads
lf1st = F1ST*XLAMD
lf = FL*XLAMD
kf = 585*QUADF/GAMMA0
# Defocusing quads
ld = FD*XLAMD
kd = -585*QUADD/GAMMA0
# Undulator focusing
L = DRL*XLAMD
Ku = K**2*ku**2/(2*GAMMA0**2)
```

```
# Thin lens approximation
f = (lf*kf)**-1
phi_c = 2*np.arcsin((lf+ld+2*L)/(4*f))
print(f"Simple phi_c={phi_c}")
print(f"Simple beta_max={{(lf+ld+2*L)*(1+np.sin(phi_c/2))/np.sin(phi_c)}}")
print(f"Simple beta_min={{(lf+ld+2*L)*(1-np.sin(phi_c/2))/np.sin(phi_c)}}")
```

Simple phi_c=0.2859559977103042
Simple beta_max=16.57388512332508
Simple beta_min=12.43970009250435

Horizontal FODO in a thick lens approximation

```
FODOx = focus(kf, lf1st) @ \
        drift(L) @ focus(kd, ld) @ drift(L) @ \
        focus(kf, lf1st)
print(FODOx)
```

```
[[ 0.96200859  4.64624157]
 [-0.01604296  0.96200859]]
```

```
phi_c = (np.arccos(np.trace(FODOx)/2))
print(phi_c)
```

```
0.27653006629258703
```

```
beta_x = FODOx[0,1]/np.sin(phi_c)
print(beta_x)
```

```
17.018003459945035
```

```
np.sqrt(beta_x*0.2e-6/GAMMA0)
```

```
1.2038964357474105e-05
```

- The phase advance is now smaller;
- The maximum of the beta function is now greater.

Vertical FODO

Without “natural focusing”

```
FOD0y = focus(-kf, lf1st) @ \
        drift(L) @ focus(-kd, ld) @ drift(L) @ \
        focus(-kf, lf1st)
print(FOD0y)
```

```
[[ 0.96200859  3.53536912]
 [-0.02108393  0.96200859]]
```

```
phi_c = (np.arccos(np.trace(FOD0y)/2))
print(phi_c)
```

```
0.2765300662925874
```

```
beta_y = FOD0y[0,1]/np.sin(phi_c)
print(beta_y)
```

```
12.949159647934803
```

```
np.sqrt(beta_y*0.2e-6/GAMMA0)
```

```
1.0501603512555186e-05
```

With “natural focusing”

```
FOD0y = focus(-kf, lf1st) @ \
        focus(Ku, L) @ focus(-kd, ld) @ focus(Ku, L) @ \
        focus(-kf, lf1st)
print(FOD0y)
```

```
[[ 0.96085158  3.53385877]
 [-0.0217225  0.96085158]]
```

```
phi_c = (np.arccos(np.trace(FOD0y)/2))
print(phi_c)
```

```
0.2807367077713572
```

```
beta_y = FOD0y[0,1]/np.sin(phi_c)
print(beta_y)
```

```
12.754684818958047
```

```
np.sqrt(beta_y*0.2e-6/GAMMA0)
```

```
1.042244688359981e-05
```

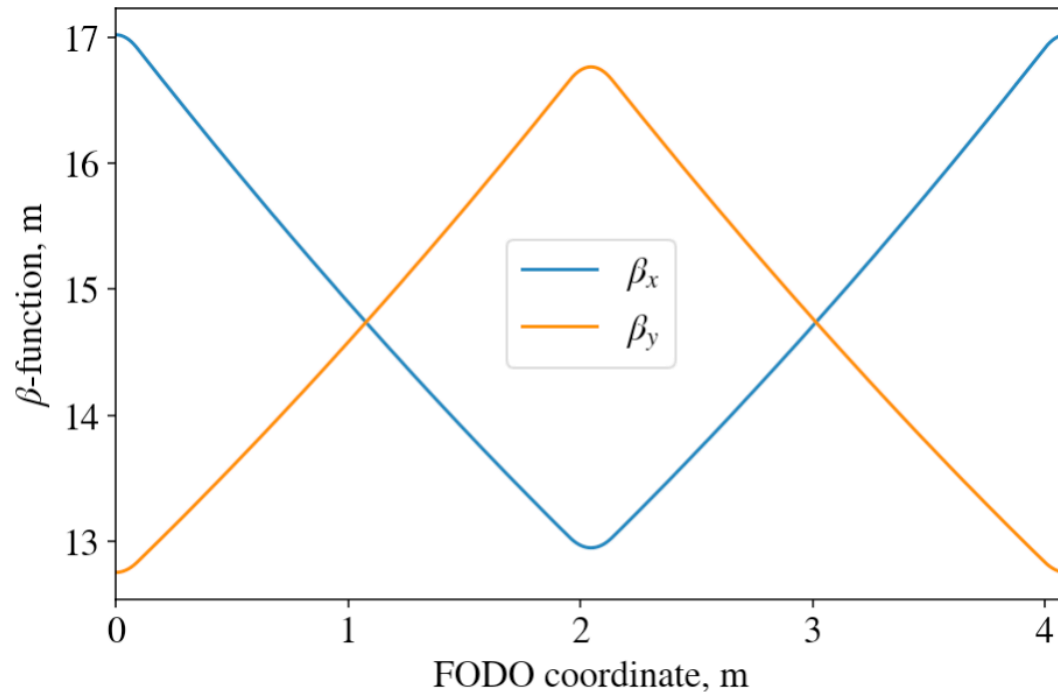
Betatron Envelope Equation

- The focusing function $K(s)$ is real and hence the amplitude and phase functions satisfy $w'' + K w - \frac{1}{w^3} = 0$, and $\psi' = \frac{1}{w^2}$, where the normalization is chosen such that w^2 is exactly the Courant-Snyder β -function and the Courant-Snyder function $\alpha = -\frac{\beta'}{2} = -w w'$.
- We want to use the numerical formalism in order to calculate the β -function in the FODO cell instead of a single point value obtained by the matrix formalism;
- We then use the numerical formalism in order to evaluate the average β -function in the FODO cell and compare this value to arithmetic average between max and min values of the β -function:

$$\bar{\beta} = \frac{\beta_{max} + \beta_{min}}{2} \quad \text{vs} \quad \bar{\beta} = \frac{1}{L_c} \int_0^{L_c} \beta(s) ds$$

Betatron envelope equation results

BetatronEnvelopeEquation.ipynb

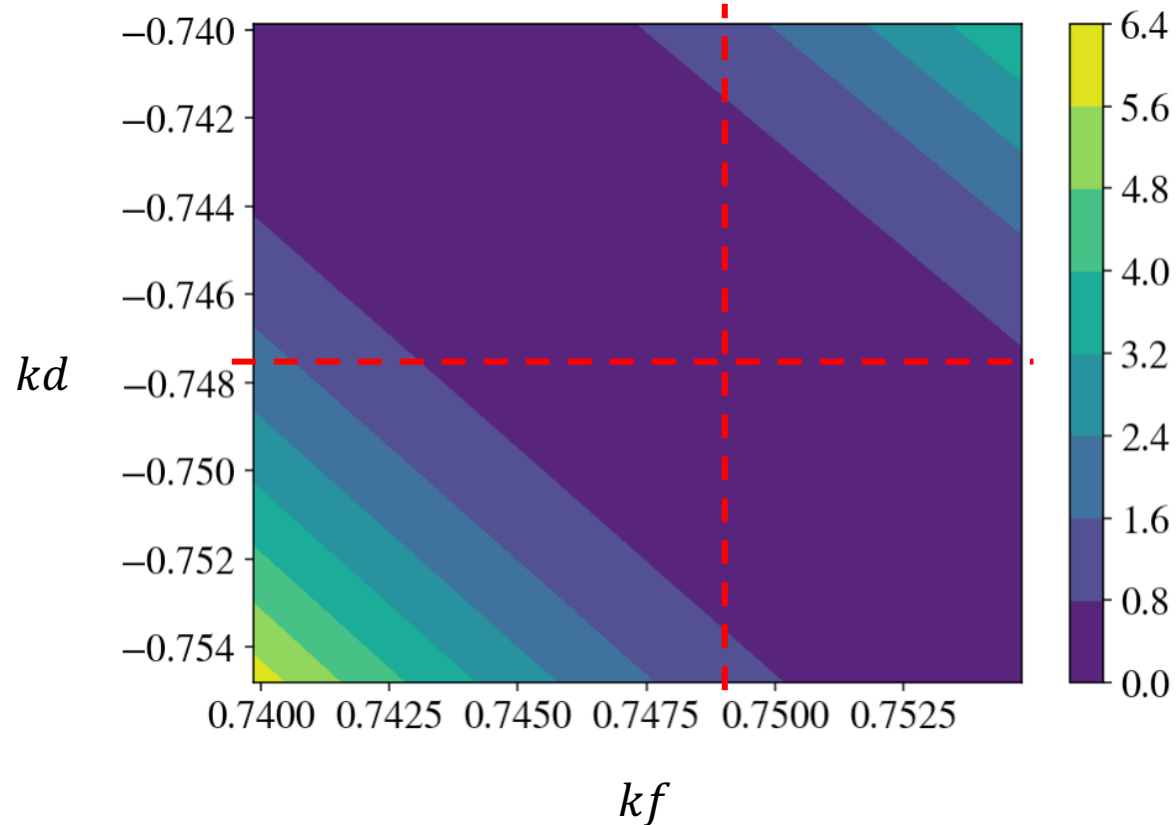


$\bar{\beta}_x = 14.898$ m and $\bar{\beta}_y = 14.675$ m are both less than the arithmetically average $\bar{\beta}$ -function.

- The thick lens solution has smooth turns inside the quadrupoles instead of sharp ones in the thin lens approximation;
- The vertical solution is focused more due to the natural focusing in the undulator;
 - Horizontal solution:
 - The maximum x beta function 17.018 m.
 - Phase advance is 0.27653 rad.
 - The minimum x beta function 12.949 m.
 - Vertical solution:
 - The maximum y beta function 12.755 m.
 - Phase advance is 0.28074 rad.
 - The minimum y beta function 16.763 m.

$\bar{\beta}$ -function matching

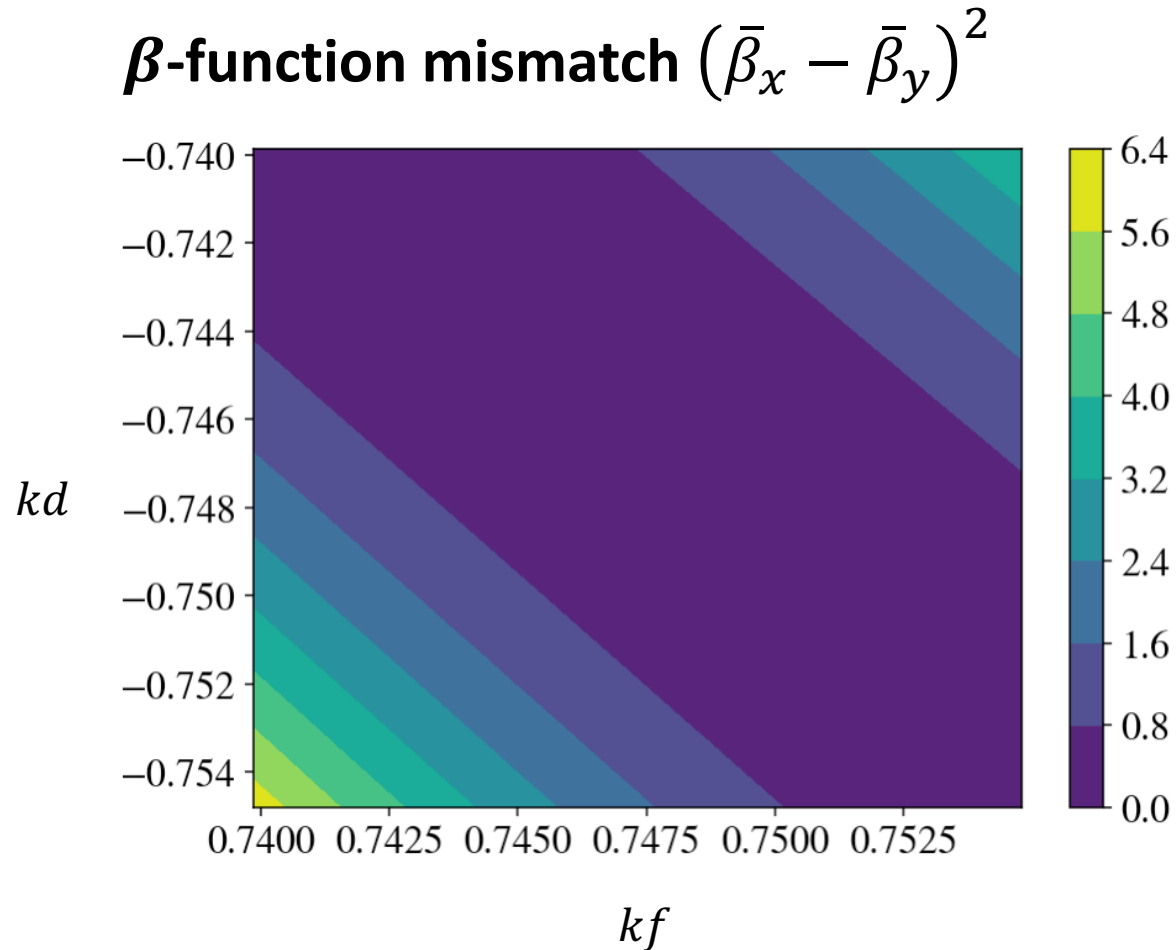
β -function mismatch $(\bar{\beta}_x - \bar{\beta}_y)^2$



- The matched solution requires a focusing quad, which defocuses in the vertical plane, to be stronger than a defocusing quad in order to compensate for the natural focusing of the undulator:

- QUADF = 30.061 T/m;
- QUADD = -30 T/m;

$\bar{\beta}$ -function matching



- Horizontal solution:
 - The phase advance is 0.27892 rad.
 - Average x beta function is 14.771 m.
 - The maximum x beta function is 16.876 m.
 - The minimum x beta function is 12.837 m.
- Vertical solution:
 - The phase advance is 0.27892 rad.
 - Average y beta function is 14.771 m.
 - The maximum y beta function is 16.874 m.
 - The minimum y beta function is 12.836 m.
- The $\bar{\beta}$ -functions are matched but the maximum and minimum are not the same due to the natural focusing in one plane modifying the beta function evolution.

Ming Xie analysis of 3D FELs

Ming Xie solution

- Ming Xie solution has been implemented in `zfel.old_scripts.mingxie()`
- Inputs keyword arguments:
 - `sigma_x` # RMS beam size
 - `und_lambda` # Undulator period (m)
 - `und_k` # Undulator K
 - `current` # Beam current (A)
 - `gamma` # Relativistic gamma
 - `norm_emit` # Normalized emittance (m-rad)
 - `sigma_E` # RMS energy spread (eV)
- Output as dictionary:
 - `gain_length` # Gain length (m)
 - `saturation_length` # Saturation length (m)
 - `saturation_power` # Saturation power (W)
 - `fel_wavelength` # FEL wavelength (m)
 - `pierce_parameter` # Pierce parameter (rho)
- My Mathematica implementation is presented here.

```

SaturationL[ $\sigma_{\gamma}$ ,  $\epsilon$ ,  $\beta$ ,  $\lambda u$ :  $1.86 \times 10^{-2}$ ,  $\lambda l$ :  $0.3 \times 10^{-10}$ ,  $bEnergy$ :  $12 \times 10^9$ ,  $Ib$ : 3000] :=
Block[{ $k_u$ ,  $k_l$ ,  $\gamma b$ ,  $K$ ,  $\xi$ ,  $JJ$ ,  $\sigma r$ ,  $IA$  = 17 050,  $\rho$ ,  $LG0$ ,  $\eta \gamma$ ,  $\eta \epsilon$ ,  $\eta d$ ,  $\Delta$ ,  $Pb$ ,  $Psat$ ,  $\alpha$ ,  $Pn$ },
(*Design Optimization for an X-ray Free Elecgron Laser Driven by SLAC Linac
Ming Xie, Lawrence Berkeley Laboratory, Berkeley, CA 94720, USA
http://accelconf.web.cern.ch/AccelConf/p95/ARTICLES/TPG/TPG10.PDF*)
 $k_u = 2 \pi / \lambda u$ ;  $k_l = 2 \pi / \lambda l$ ;
 $\gamma b = 1 + bEnergy / 510\,999.$ ;
 $K = \sqrt{2} \sqrt{2 \gamma b^2 \lambda l / \lambda u - 1}$ ;
 $\xi = \frac{K^2}{4 + 2 K^2}$ ;  $JJ = BesselJ[0, \xi] - BesselJ[1, \xi]$ ;
 $\sigma r = \sqrt{\epsilon \beta / \gamma b}$ ;
 $\rho = \left( \frac{1}{16} \frac{Ib}{IA} \frac{K^2 JJ^2}{\gamma b^3 \sigma r^2 k_u^2} \right)^{1/3}$ ; (*FEL parameter*)
 $LG0 = \frac{\lambda u}{4 \pi \sqrt{3} \rho}$ ; (*1D Gain Length*)
(*Ming Xie Parametrization*)
 $\eta d = \frac{LG0}{2 k_l \sigma r^2}$ ;  $\eta \epsilon = LG0 \frac{(4 \pi) \epsilon / \gamma b}{\beta \lambda l}$ ;  $\eta \gamma = 4 \pi \frac{LG0}{\lambda u} \sigma \eta$ ;
 $\Delta = 0.45 \eta d^{0.57} + 0.55 \eta \epsilon^{1.6} + 3 \eta \gamma^2 + 0.35 \eta \epsilon^{2.9} \eta \gamma^{2.4} + 51 \eta d^{0.95} \eta \gamma^3 + 5.4 \eta d^{0.7} \eta \epsilon^{1.9} + 1140 \eta d^{2.2} \eta \epsilon^{2.9} \eta \gamma^{3.2}$ ;
(*Conclusions*)
 $Pb = Ib bEnergy$ ;
 $Psat = 1.6 \rho Pb / (1 + \Delta)^2$ ;
 $\alpha = 1 / 9$ ;
 $Pn = \rho^2 bEnergy 1.60217662 \times 10^{-19} \times 299\,792\,458 / \lambda l$ ;
Return[ $LG0 (1 + \Delta) \text{Log}[Psat / Pn / \alpha]$ ];
];

```

Ming Xie evaluation – single value

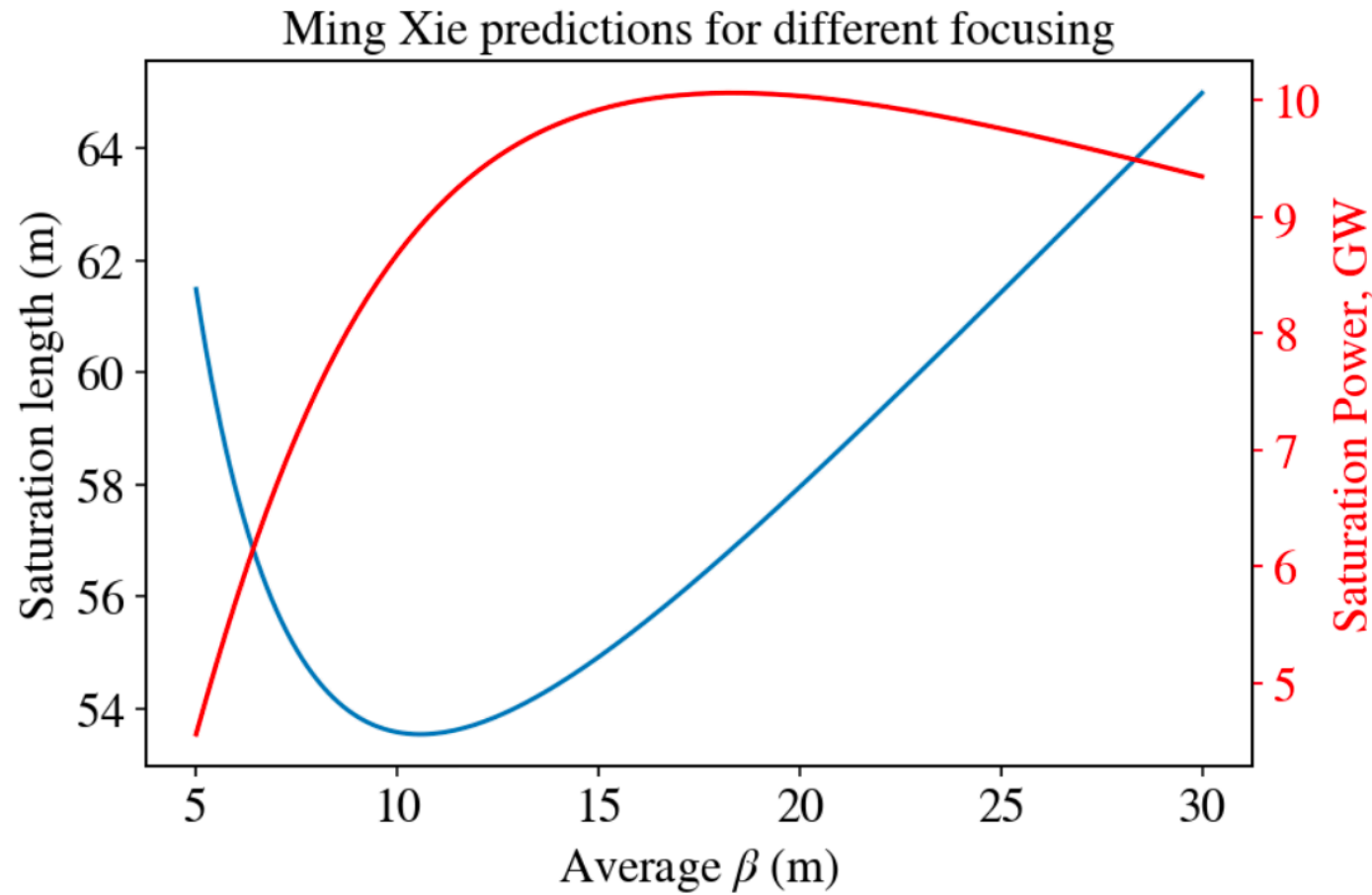
```
XLAMD = 1.86e-2 # undulator period
XLAMDS = 0.3e-10 # desired x-ray wavelength
CURPEAK = 3e3 # peak current
GAMMA0 = 12e9/0.511e6 # energy in mc2
energy = 0.511e6*GAMMA0 # beam energy in eV
DELGAM = 1.5e-4
sigma_e = DELGAM*energy # energy spread in eV
AWO = np.sqrt(2*GAMMA0**2*XLAMDS/XLAMD-1) # we use resonant condition formula here
K = np.sqrt(2)*AWO
EMITX = 0.2e-6 # normalized emittance in x
beta = 15 # beta function in meters
RXBEAM = np.sqrt(beta*EMITX/GAMMA0) # the corresponding beam size
```

```
# Some parameters
params = {
    'sigma_x':RXBEAM,
    'und_lambda':XLAMD,
    'und_k':K,
    'current':CURPEAK,
    'gamma':GAMMA0,
    'norm_emit':EMITX,
    'sigma_E':sigma_e}
```

```
mingxie(**params)
```

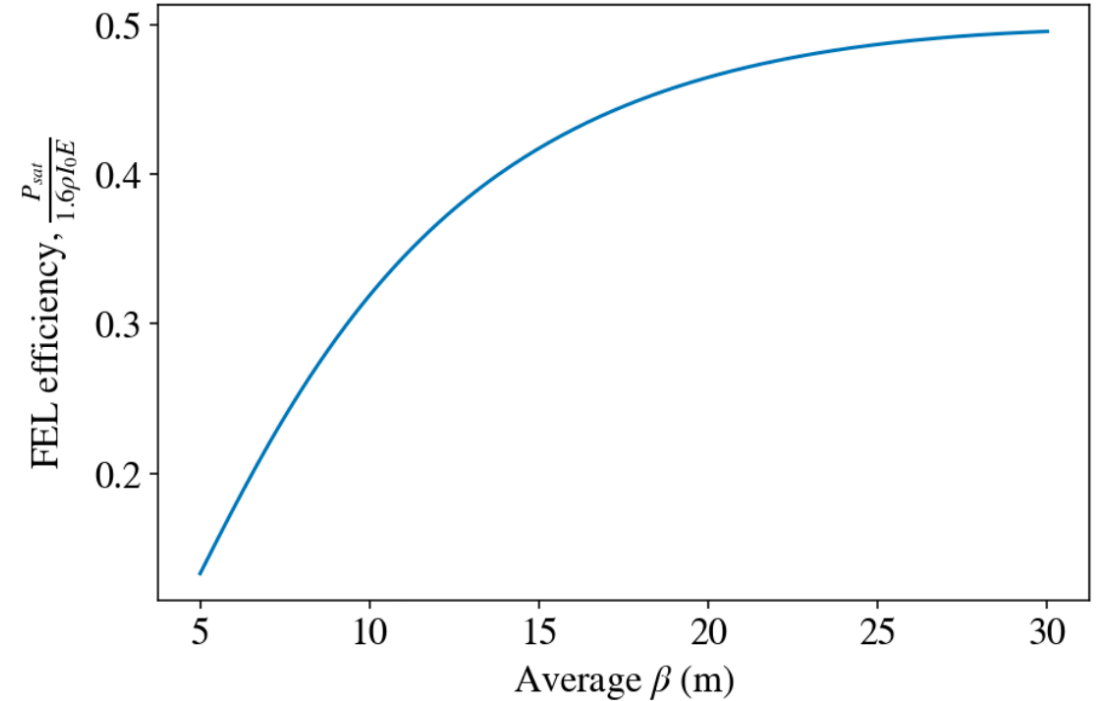
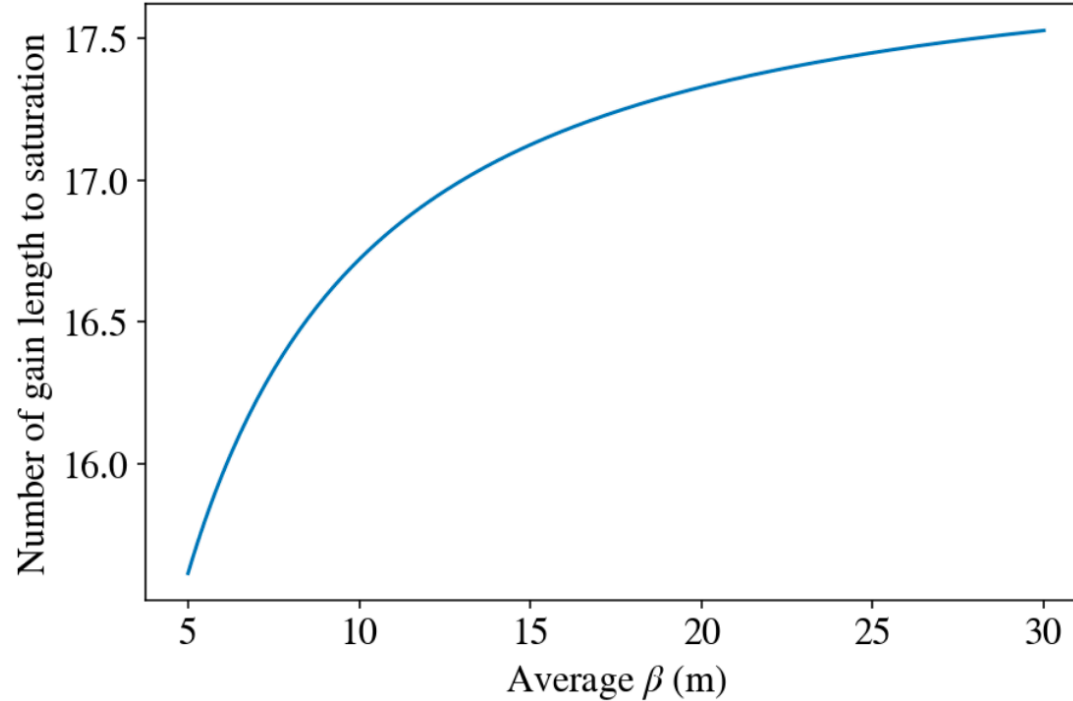
```
{'gain_length': 3.21,
'saturation_length': 54.91,
'saturation_power': 9.916e9,
'fel_wavelength': 3e-11,
'pierce_parameter': 4.125e-4}
```

Ming Xie evaluation – scan



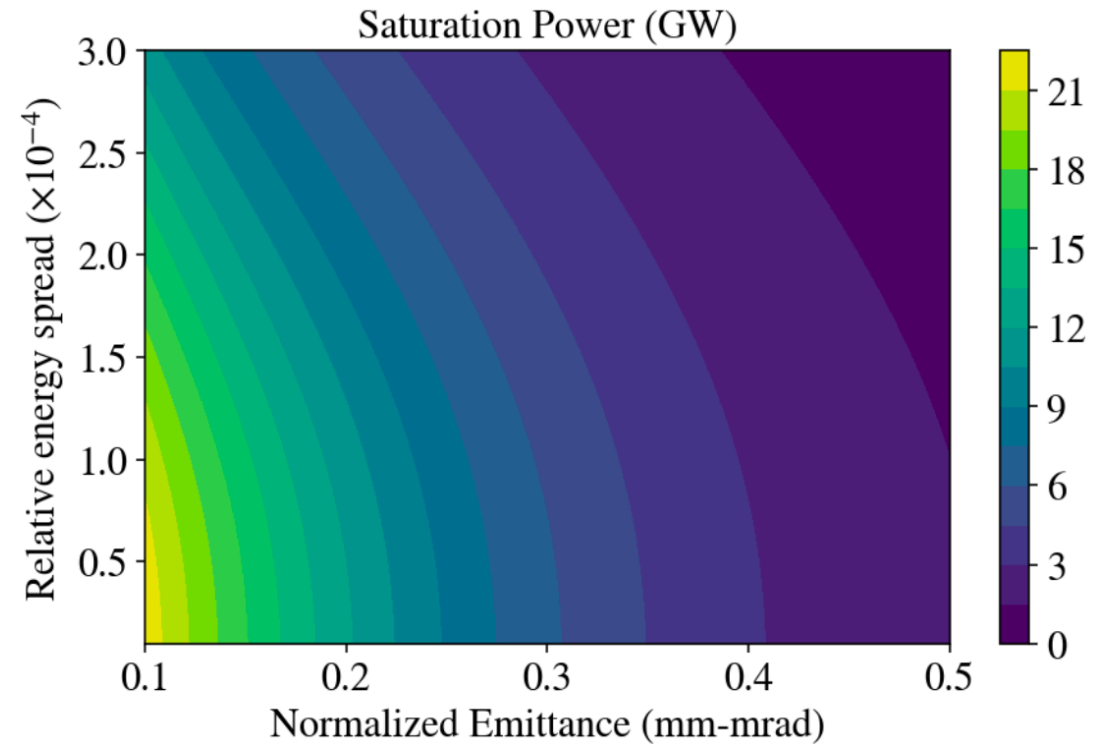
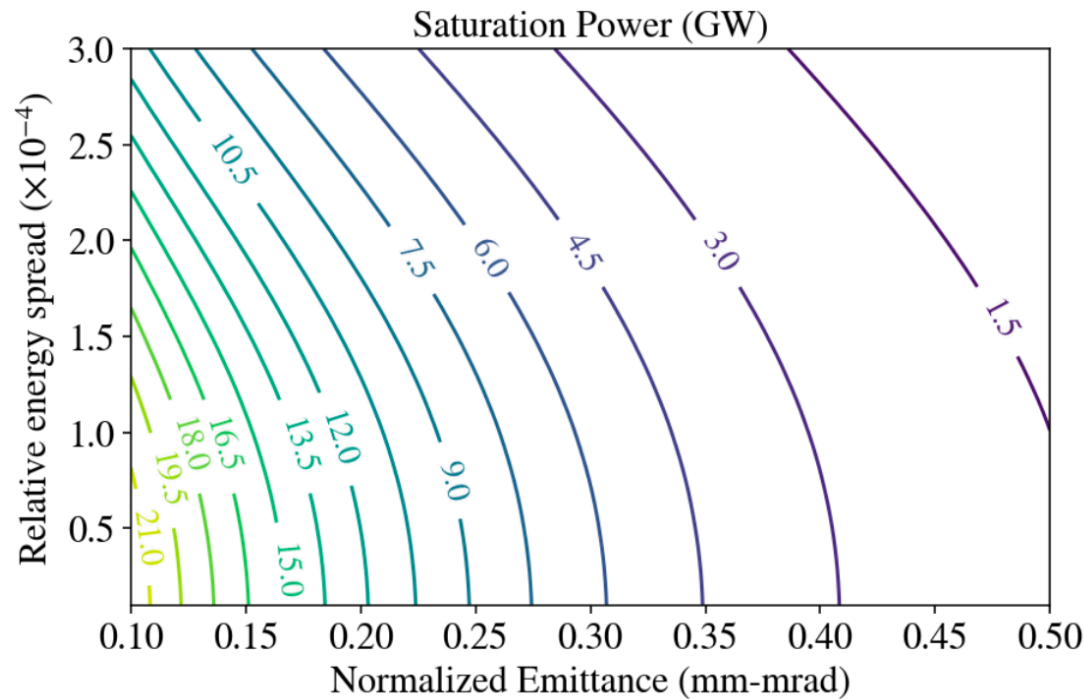
- We can use Ming Xie predictions in order to find the optimum beam size for MaRIE-like x-ray FEL;
- We can see that, although $\bar{\beta} = 10$ m results in a shorter saturation length (higher gain), the highest saturated power is reached at $\bar{\beta} = 15$ m;
- Picking up $\bar{\beta}$ provides guidance on the FODO lattice design, which we discussed in the previous section;
- Please recall the value of the previously discussed $\bar{\beta}$ values; they were closer to the power maximum than to the gain maximum, which is inversely related to the saturation length.

Ming Xie evaluation – scan



- The saturation length measured in the number of gain length is less than predicted by 1D theory;
- The saturated power is less than 50% of that predicted by 1D theory;
- These degradations are due to the 3D effects.

ZFEL execution – 2 variable scan



- $\bar{\beta} = 15 \text{ m}$ case is studied here;
- This analysis shows that XFEL performance is highly sensitive to emittance and energy spread;
- The studied design here is emittance dominated and one would require a low emittance beam!

3D Numerical simulator – LUME-Genesis

Genesis v2

- We will focus on Genesis v2, which is a stable version of a very popular FEL modeling code (<http://genesis.web.psi.ch/>) written in Fortran;
- It uses undulator averaged approximation and thus expresses all the distances in the units of undulator period XLAMD;
- Electromagnetic fields are expressed on the Cartesian grid;
- Electrons are represented by an equal number of macroparticles arranged in slices, one resonant wavelength XLAMDS long;
- Slices are ZSEP wavelength apart;
- LUME-genesis is python interface to setup, run and analyze Genesis v2 simulations;
- Genesis v4, rewritten in C, is under active development now (<https://github.com/svenreiche/Genesis-1.3-Version4>);

Genesis v2: Input file <http://genesis.web.psi.ch/Manual/files1.html>

- Concise description of the input parameters could be found in <https://github.com/slaclab/Genesis-1.3-Version2/blob/master/input.f#L1272> as well as <https://github.com/ocelot-collab/ocelot/blob/master/ocelot/adaptors/genesis.py#L202>
- Genesis generates 'template.in' if ran without an input file provided!

Congratulations!

This is your first Genesis simulation!

```
plot as plt
'data'[['z']], gen.output['data']['power'][0]
plt.ylabel('Power (GW)');
```

```
!genesis2
```

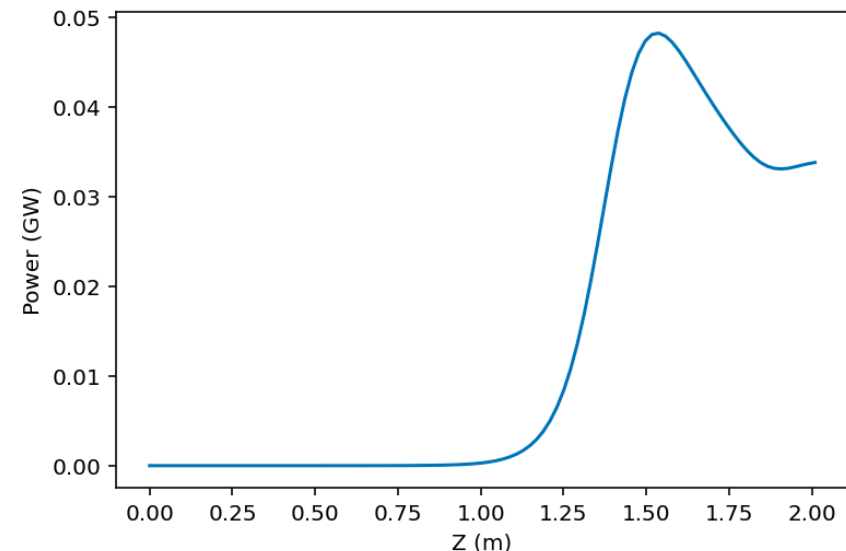
```
-----
Genesis 1.3 has begun execution
(Version 2.3 Unix)
```

```
Please enter input file name
```

```
*** File-error:
*** cannot be opened
*** creating template file: template.in
*** closing files
```

```
Genesis run has finished
-----
```

```
from genesis import Genesis
gen = Genesis('template.in')
gen.run()
```



Genesis v2: MaRIE input file

```
genesis_bin='/home/vagrant/.local/bin/genesis2-mpi'
gen = Genesis('template.in', genesis_bin=genesis_bin)
gen.binary_prefixes = ['mpirun', '-n', '4']

# undulator
gen['xlamd'] = 0.0186 # undulator wavelength, m
gen['aw0'] = gen['awd'] = 0.86 # rms undulator parameter
gen['nwig'] = int(80/gen['xlamd']) # undulator length in xlamd

# focusing
gen['f1st'] = 5 # half F length in FODO measured in xlamd
gen['fl'] = 10 # full F length in FODO measured in xlamd
gen['quadf'] = 30 # focusing in x quadrupole gradient, T/m
gen['dl'] = 10 # full D length in FODO measured in xlamd
gen['quadd'] = 30 # defocusing in x quadrupole gradient, T/m
gen['drl'] = 100 # full O length in FODO measured in xlamd

# electron beam
gen['curpeak'] = 3000 # current, A
gen['curlen'] = 0 # negative for flattop; positive for Gaussian
gen['gamma0'] = 12e9/0.511e6 # beam energy, mc^2
gen['delgam'] = 1.5e-4*gen['gamma0'] # relative energy spread
gen['rxbeam'] = 1.2038964357474105e-05 # rms size, m
gen['rybeam'] = 1.042244688359981e-05 # rms size, m
gen['emitx'] = gen['emity'] = 0.2e-6 # normalized emittance, m rad
gen['npart'] = 2**10 # number of macroparticles in a bucket
```

```
# radiation at resonant condition
gen['xlamds'] = gen['xlamd']*(1+gen['aw0']**2)/(2*gen['gamma0']**2)
gen['prad0'] = 1e4 # shot noise power, W
gen['zrayl'] = 24 # Rayleigh length, m
gen['zwaist'] = 0 # focal point location, m

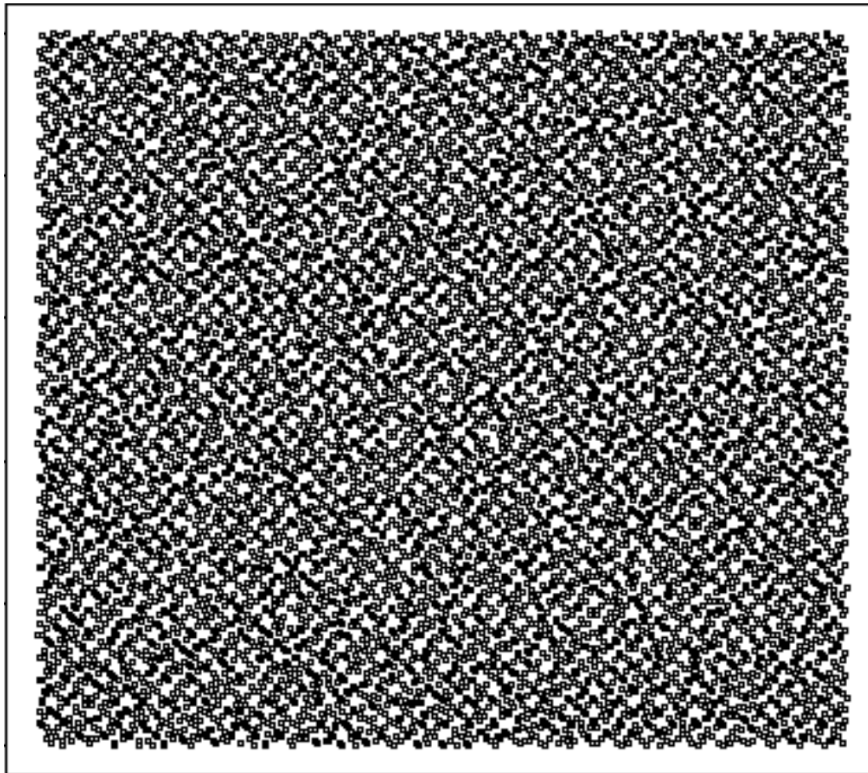
# mesh
gen['ncar'] = 151 # number of mesh points, ODD is advised
gen['dgrid'] = 100e-6 # [-dgrid, dgrid], m

# simulation
gen['delz'] = 1 # integration step measured in xlamd
gen.run()
gen.output['run_info']

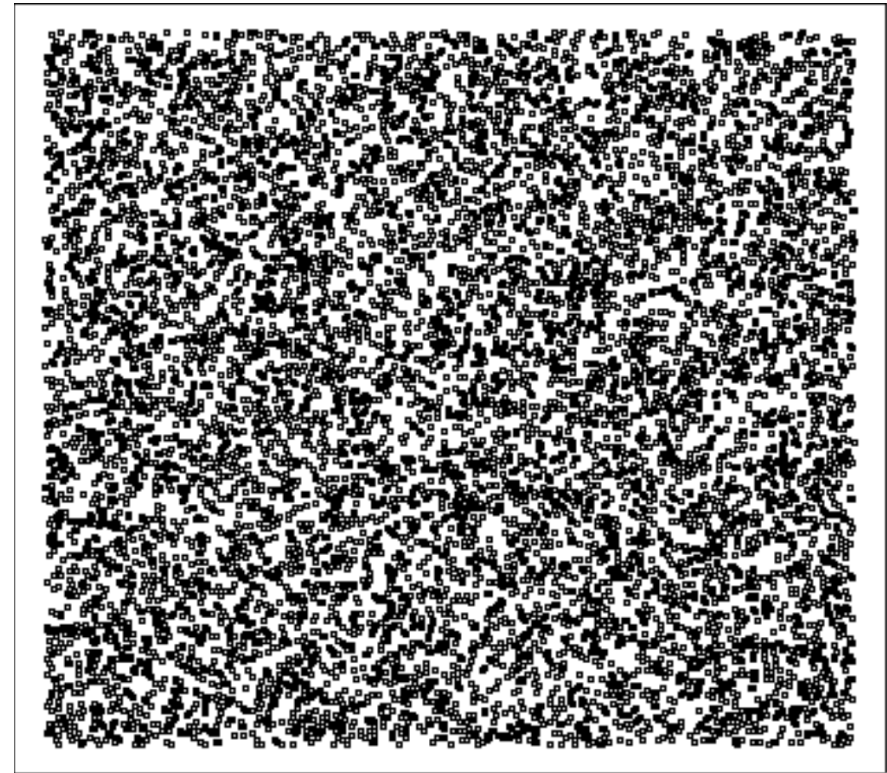
{'start_time': 1611354311.8145232,
 'run_script': 'mpirun -n 4 /home/vagrant/.local/bin/genesis2-mpi ge
 nesis.in',
 'run_time': 6.585921049118042,
 'run_error': False}
```


Particle loading – ‘quiet start’

The first 10000 points in the same sequence. These 10000 comprise the first 1000, with 9000 more points.



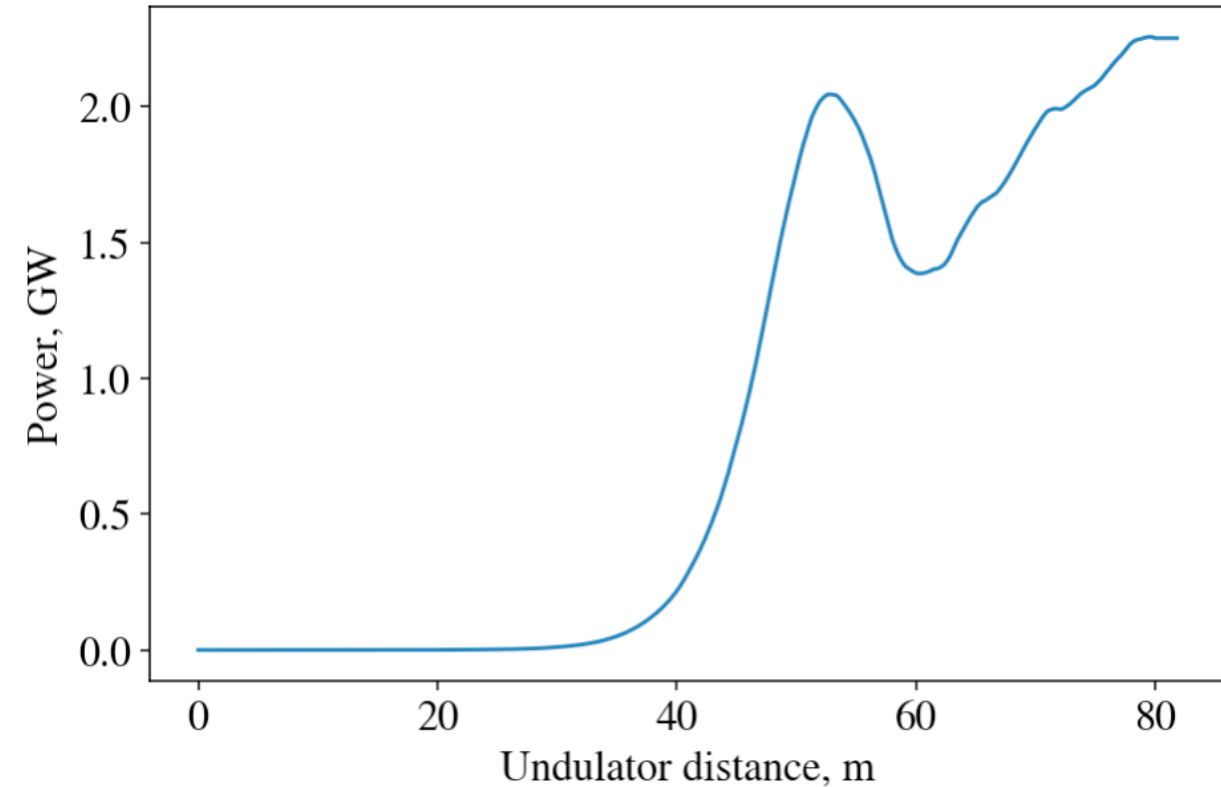
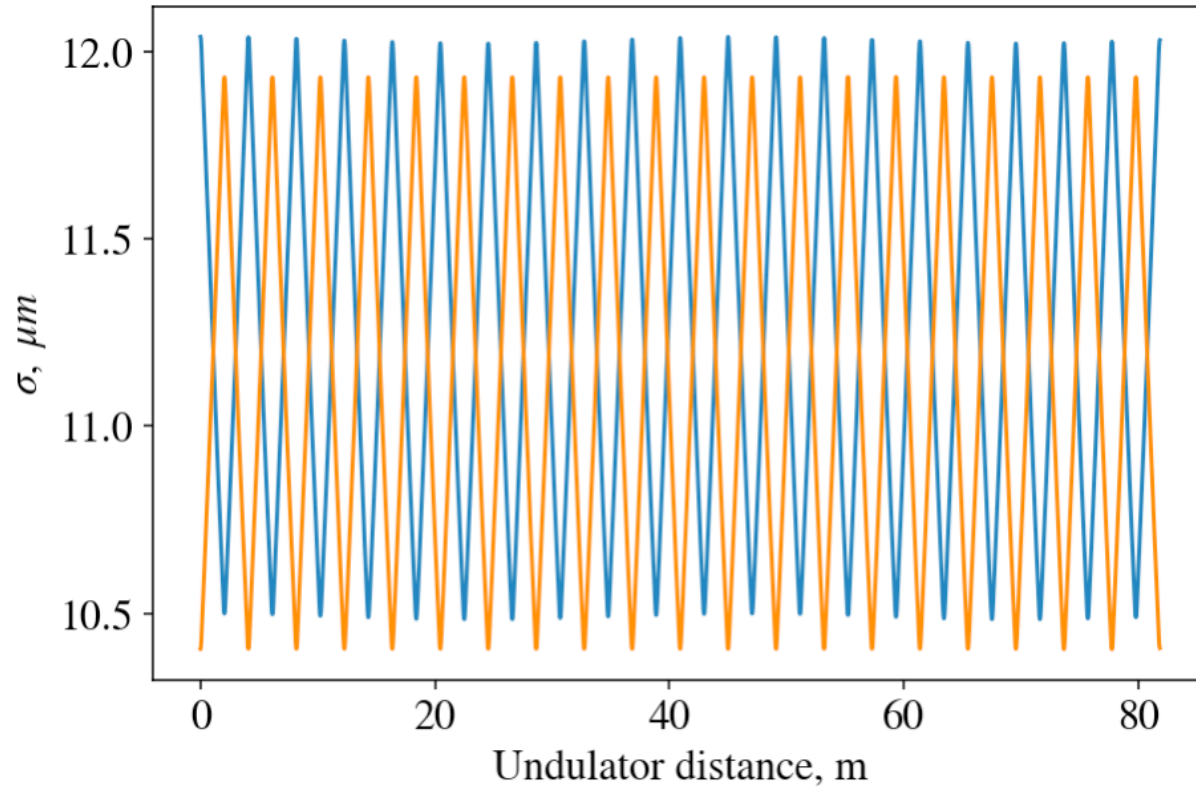
The first 10000 points in a sequence of uniformly distributed pseudorandom numbers. Regions of higher and lower density are evident.



Genesis uses Hammersley sequence for quiet start instead of pseudorandom numbers that introduces bunching!

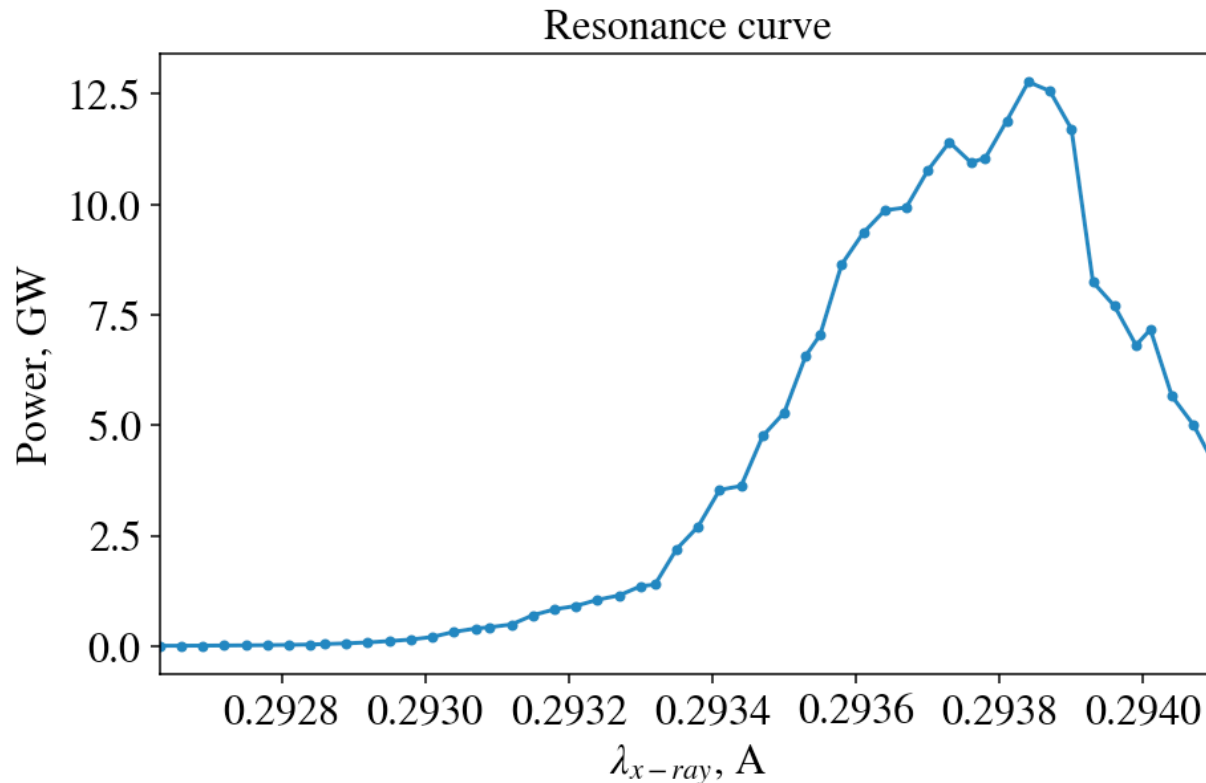
https://en.wikipedia.org/wiki/Low-discrepancy_sequence#Hammersley_set

Results of the no SASE simulation



Electron beam size is FODO matched but not $\bar{\beta}$ matched. Generated power is less than expected from Ming Xie. **Why?**

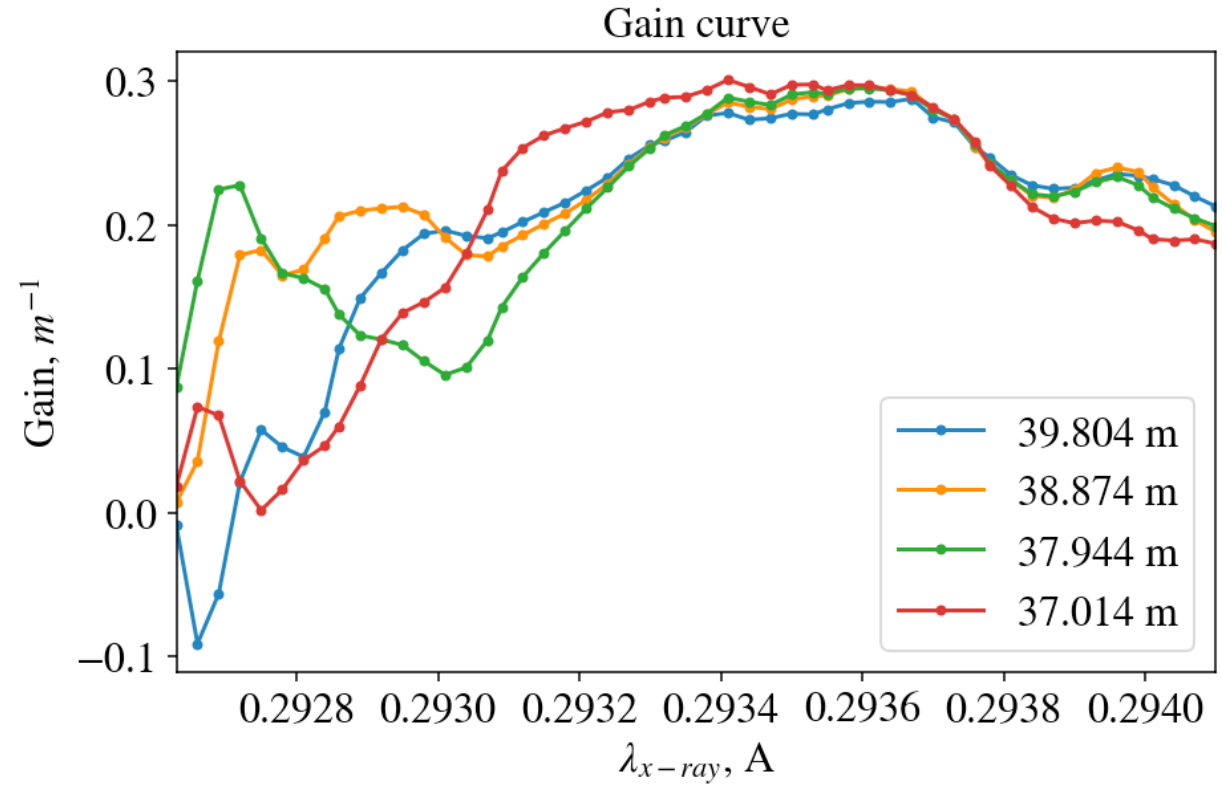
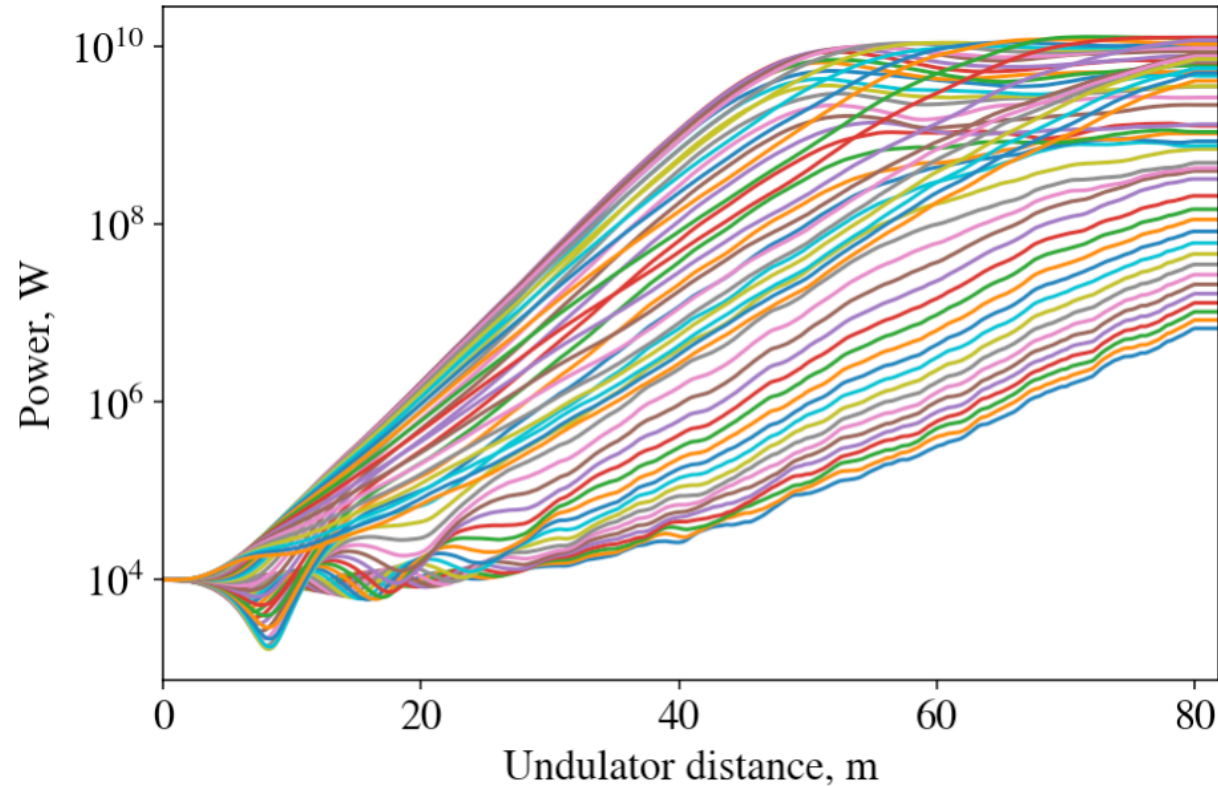
Power optimization



```
gen['iscan'] = 4
gen['nscan'] = 52
gen['svar'] = 0.0025
gen.run()
gen.output['run_info']
```

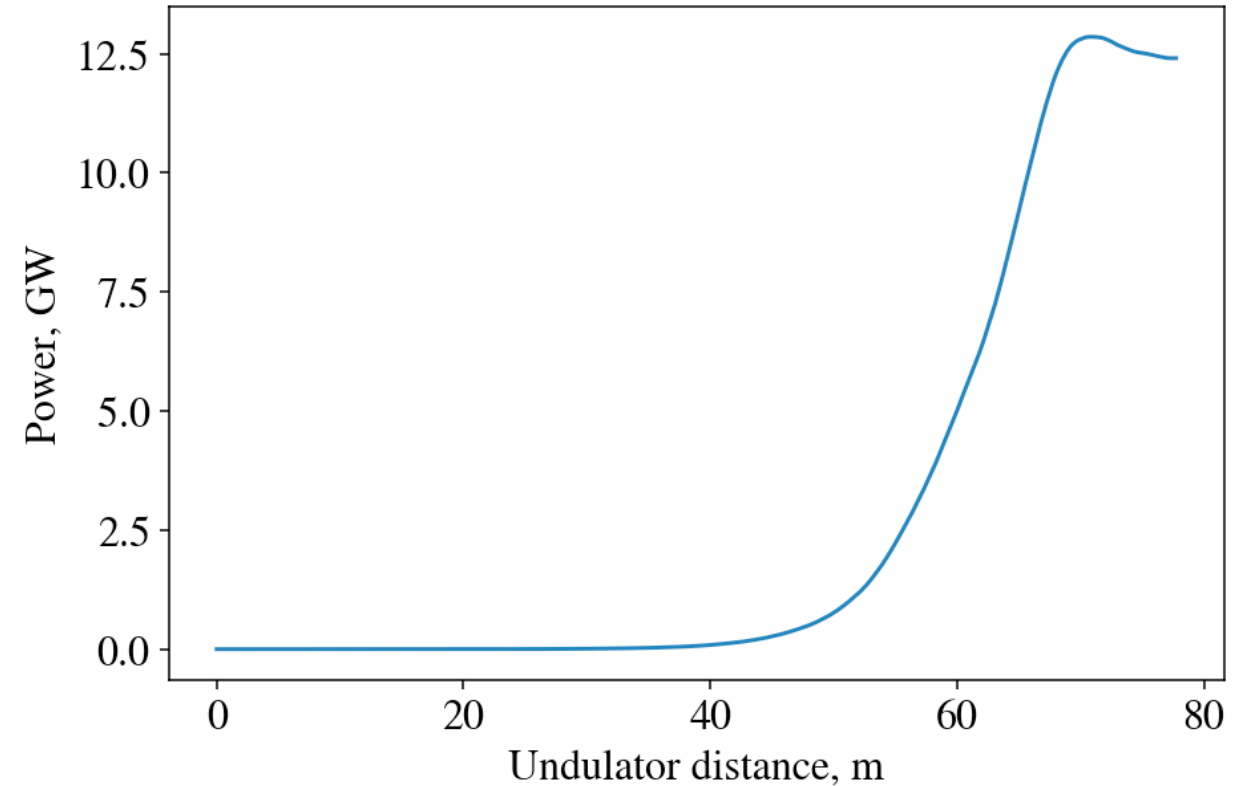
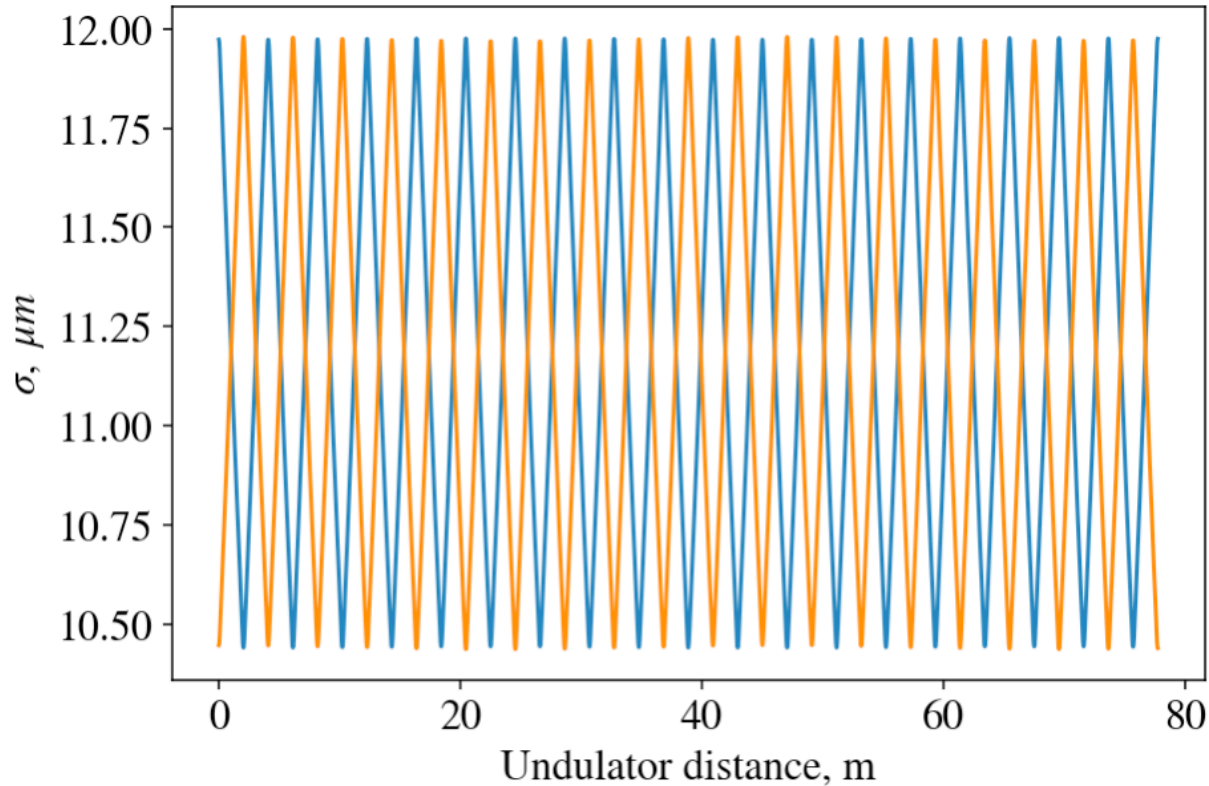
- The steady state simulation keeps the radiation wavelength fixed in order to determine the size of the slice;
- This type of simulation assumes no slippage of the radiation with respect to the electrons;
- Any electrons that fall behind get reintroduced into the slice assuming that an identical slice exists ahead of the simulated slice;
- The expected wavelength was $2.9337\text{e-}11$ m but we have found $2.9384\text{e-}11$ m, which is 0.0016 relative shift.

Gain optimization



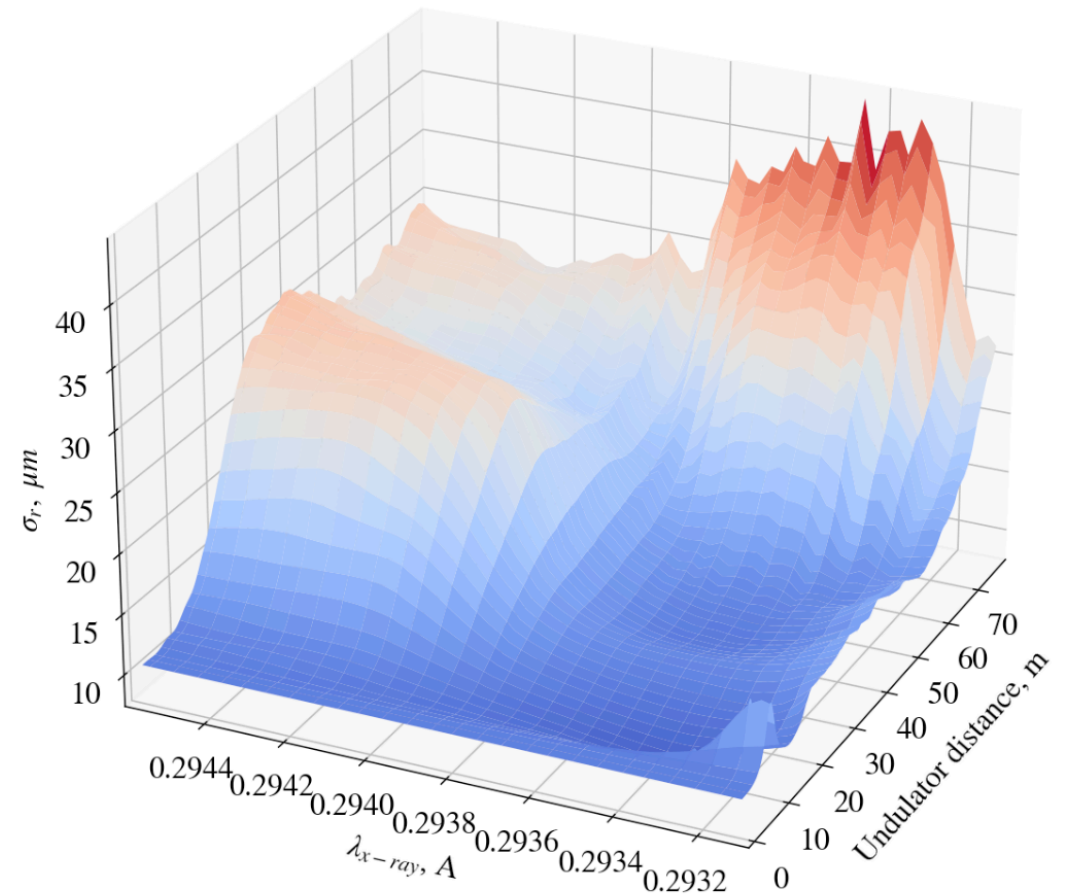
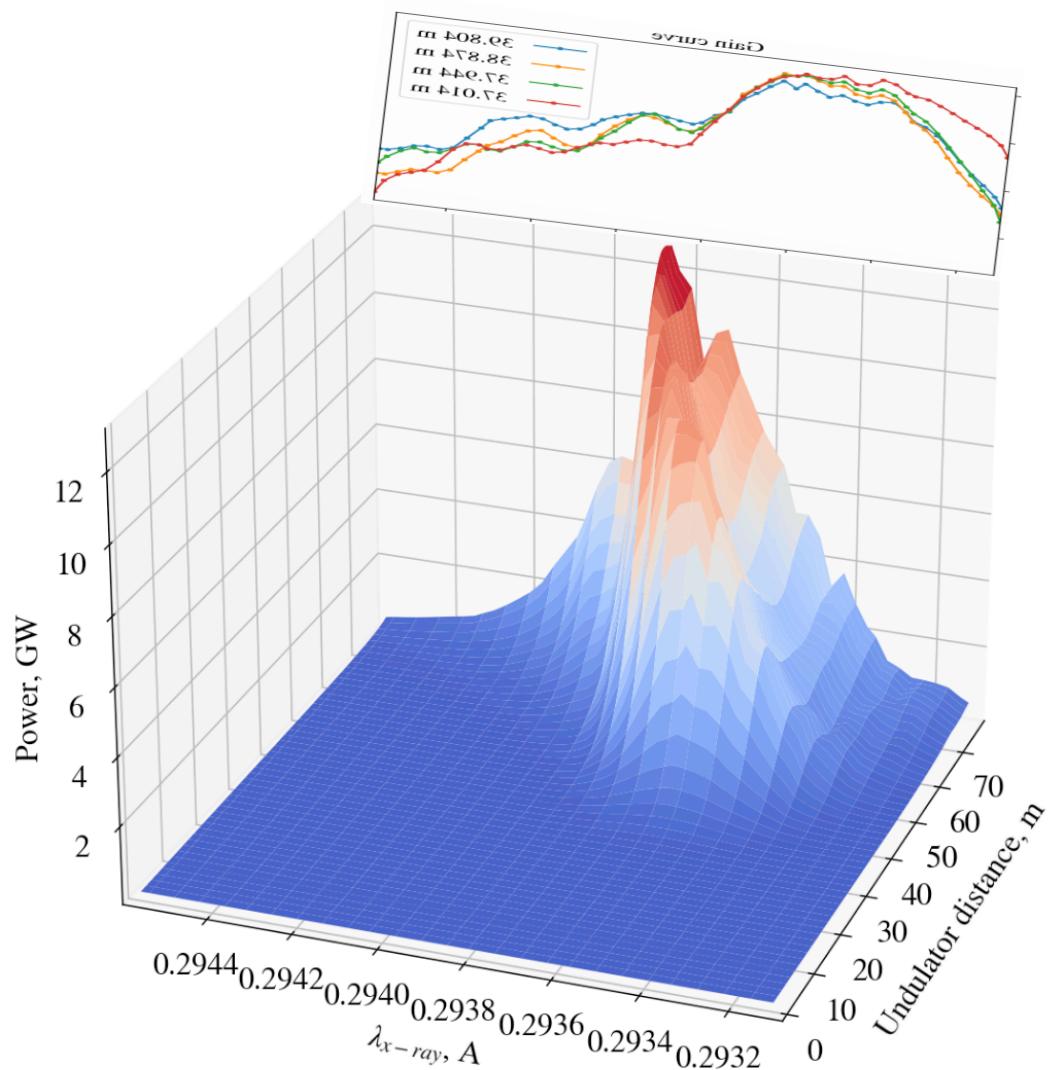
We look at the power growth along the undulator and see oscillations at the start;
We then estimate the Gain coefficient at the several points along the undulator;
The gain peaks at the resonant wavelength and not where the power picked!

$\bar{\beta}$ -function matching



$\bar{\beta}$ -function matched solution produces a round beam in the undulator but not necessary the highest power!
 Keeping a round beam may allow a smaller undulator gap and a stronger undulator parameter;
 Our solution saturates at a longer undulator distance than Ming Xie predicted.

Gain guiding of the optical mode



Genesis v2: Lattice file <http://genesis.web.psi.ch/Manual/files6.html>

```
outmagfile = 'file_name'
delz        = 5
version     = 1.0
```

[illegible]

QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	2	20
QF	-3.0000E+01	2	20
QF	3.0067E+01	1	20
QX	0.0000E+00	836	0
QY	0.0000E+00	836	0
AD	0.0000E+00	836	0
SL	0.0000E+00	836	0
CX	0.0000E+00	836	0
CY	0.0000E+00	836	0
AX	-0.0000E+00	836	0
AY	-0.0000E+00	836	0

Genesis v2: Lattice file <http://genesis.web.psi.ch/Manual/files6.html>

- Header of the file:
 - ? VERSION=1.0
 - ? UNITLENGTH='xx'
- Element line:
 - 'XX' 'strength' 'length' 'offset'
- A two-character string, 'XX', indicating the type of structure. Following types are supported:
 - AW - Main magnetic field
 - AD - Drift section
 - QF - Quadrupole strength
 - QX - Quadrupole offset in x; QY - Quadrupole offset in y
 - SL - Solenoid strength
 - CX - Corrector strength in x; CY - Corrector strength in y

Genesis v2: Lattice file simplified

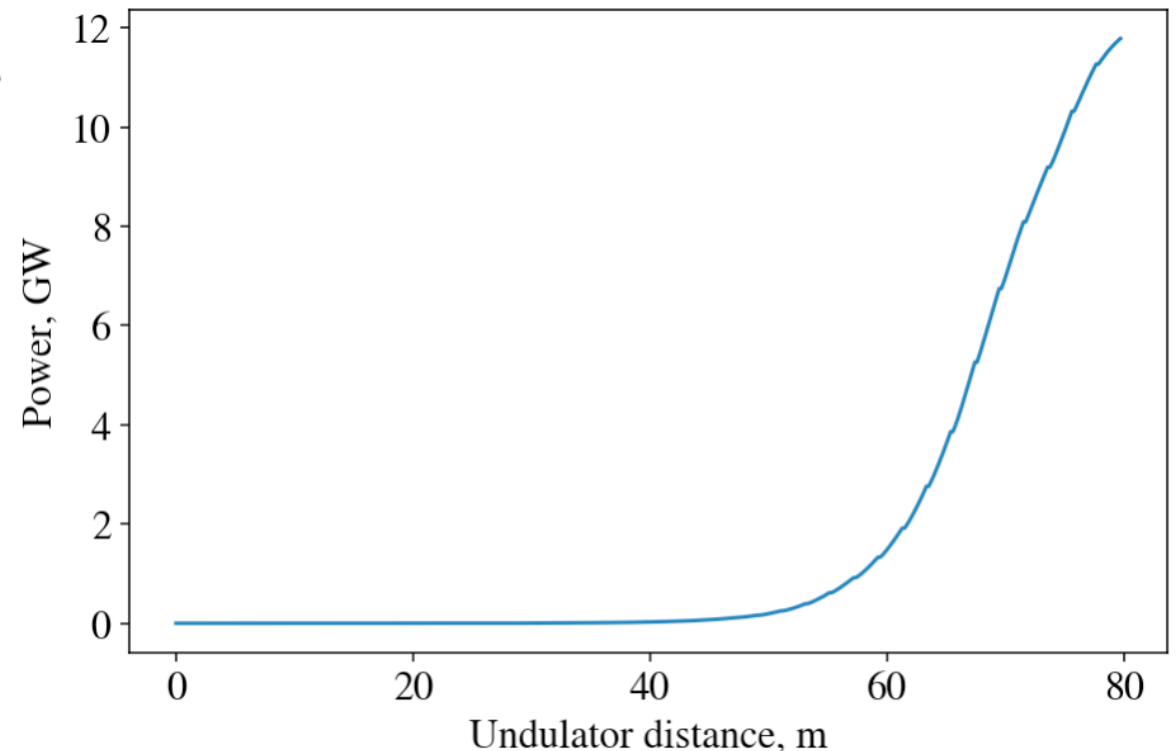
- `maginfile='marie.lat'` supersedes the input file values yet it also depends on

- A real FEL has undulator sectioned!

- Let us introduce the gaps in sections:

```
# header is included
? VERSION= 1.00 including new format
? UNITLENGTH= 0.09300 :unit length in head
QF      3.0067E+01      1      0
AW      8.6000E-01     20      1
! LOOP= 19
QF     -3.0000E+01      2     20
AW      8.6000E-01     20      2
QF      3.0067E+01      2     20
AW      8.6000E-01     20      2
! ENDLLOOP
```

```
gen['xlamd'] = 1.86e-2
gen['awd'] = 0.86
gen['maginfile'] = 'marie.lat'
gen.load_lattice('marie.lat')
```




```
? VERSION= 1.00  including new format
? UNITLENGTH= 0.09300 :unit length in header
```

[illegible]

Phase synchronism in drifts

- We have to fill in the gaps in order to maintain phase synchronism controlled by appropriate phase shifters in actual FELs;
- Sometimes detuning the phase shifter can be used to suppress the fundamental harmonic.

